Minimal Supersymmetry, Supersymmetric Frameworks, and the Expected Neutralino-Quark Scattering Cross-Section

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Abstract

This document summarizes supersymmetric extensions to the Standard Model, describes various frameworks in use, discusses in detail the neutralino sector, and presents expected neutralino-nucleon scattering cross-sections calculated using P. Gondolo's DarkSUSY code, with some explanation of the allowed parameter space.

1 Introduction and Motivation

This note is an extremely pedagogical introduction to supersymmetry and its applications to dark matter intended for experimenters new to the field – essentially, we hope to explain, to experimenters, why the expected neutralino WIMP parameter space is what it is. We present material at a level appropriate for readers who have taken a typical graduate particle physics course. We aspire neither to provide the theoretical rigor of the numerous "Introduction to Supersymmetry" articles available nor the calculational details needed by the practicing theorist. The idea here is to motivate and present the Lagrangian of the Minimal Supersymmetric Standard Model, including supersymmetry- and electroweak symmetry-breaking terms, to discuss what is typically assumed when making calculations, and to explore in some detail the parameter space available for neutralino WIMPs. We hope to give the reader a basic understanding of the *language* of supersymmetry and to discuss how the neutralino-quark elastic-scattering cross-section depends on assorted SUSY parameters.

This note is the result of discussions between Paolo Gondolo and the authors during his visit to the Center for Particle Astrophysics in October, 1999. We thank him for his indulgence and patience in explaining the MSSM and the neutralino's place in it to us, for defining the various calculational fraweworks, for providing his calculations of allowed neutralino parameter space, and for setting up his DarkSUSY code at the CfPA.

2 The Standard Model

We begin by reminding the reader of the particle content and Lagrangian for the Standard Model. For simplicity, we assume the neutrinos are massless and no right-handed neutrinos exist. We use a basis that is $SU(2)_{EW} \times U(1)_Y$ diagonal. That is, the fields are eigenstates of the electroweak interaction rather than mass eigenstates. In such a basis, the particle content is as follows:

2.1 Particle Content

particle content of SM, similar to table 11 of JKG don't forget to define both ϕ and $\phi^\dagger$.

2.2 Lagrangian

We begin with electroweak symmetry unbroken: all the fermions and gauge bosons are massless and the gauge symmetry group is $SU(2)_{EW} \times U(1)_Y$. The unbroken Lagrangian can be split up into a number of simple pieces as follows:

- The fermion kinetic term, modified according to the minimal coupling prescription ($\partial^\mu \to D^\mu = \partial^\mu - igA^\mu$) to yield the gauge interactions. We remind the reader that, in this basis, the matrices describing the fermion interactions with the charged W bosons are diagonal because the basis states are eigenstates of the EW interaction (j runs over generations):

$$\begin{aligned}
\mathcal{L}_{\overline{f}f,\overline{f}Vf} = \sum_j \Big[& \overline{l_{L,j}}\, i\gamma^\mu \left(\partial_\mu - ig_1 \tfrac{Y_{l_{L,j}}}{2} B_\mu - ig_2 \tfrac{\tau_a}{2} W^a_\mu\right) l_{L,j} \\
& + \overline{e_{R,j}}\, i\gamma^\mu \left(\partial_\mu - ig_1 \tfrac{Y_{e_{R,j}}}{2} B_\mu\right) e_{R,j} \\
& + \overline{q_{L,j}}\, i\gamma^\mu \left(\partial_\mu - ig_1 \tfrac{Y_{q_{L,j}}}{2} B_\mu - ig_2 \tfrac{\tau_a}{2} W^a_\mu\right) q_{L,j} \\
& + \overline{u_{R,j}}\, i\gamma^\mu \left(\partial_\mu - ig_1 \tfrac{Y_{u_{R,j}}}{2} B_\mu\right) u_{R,j} \\
& + \overline{d_{R,j}}\, i\gamma^\mu \left(\partial_\mu - ig_1 \tfrac{Y_{d_{R,j}}}{2} B_\mu\right) d_{R,j} \Big]
\end{aligned} \tag{1}$$

  B^μ is the gauge field associated with the $U(1)_Y$ symmetry and W^μ_a are the gauge fields ($a = 1, 2, 3$) associated with the $SU(2)_{EW}$ symmetry. The left-handed fields are $SU(2)_{EW}$ doublets, carrying $SU(2)_{EW}$ charge. In the terms with B_μ, the $SU(2)_{EW}$ indices of the two doublets in the term are just contracted over; in the terms with W_μ, the τ matrices are 2×2 $SU(2)_{EW}$ matrices that make use of the $SU(2)_{EW}$ structure of the left-handed fields. The right-handed fields carry no $SU(2)_{EW}$ charge and hence no $SU(2)_{EW}$ structure. Recall that B_μ and the W^a_μ mix to yield the photon A^μ, the neutral weak gauge boson Z^0_μ, and the charged weak gauge bosons, $W^\pm_\mu$. We do not include the $SU(3)_C$ interaction because it will play no interesting part in the following. g_1 and g_2 mix to yield the electomagnetic and weak coupling constants α_1 and α_2. Note also that the above makes explicit the fact that only left-handed fermions carry $SU(2)_{EW}$ charge. In this unbroken form, the left- and right-handed fermions truly appear to be different particles – they do not interact with one another directly. This is a useful perspective to take when it comes time to supersymmetrize.

- The gauge field kinetic term. The standard prescription for this is, given a gauge field A^a_μ, as follows: First, construct the field strength $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \alpha f_{abc} A^a_\mu A^b_\nu$ where f_{abc} are the structure constants for the symmetry group's generators (*e.g.*, $f_{abc} = \epsilon_{abc}$ for $SU(2)$). Then, the kinetic term is $-\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a$, where the sum over a is implied. This yields the obvious terms in the Lagrangian. We will not be concerned with them, so we do not write them out explicitly here.

- The Higgs scalar potential – designed to yield electroweak symmetry breaking (EWSB) (recall that ϕ is a $SU(2)_{EW}$ doublet):

$$\mathcal{L}_\phi = -\mu^2 \phi^\dagger \phi + \lambda \left(\phi^\dagger \phi\right)^2 \tag{2}$$

  This potential has a minimum at $\phi^\dagger \phi = -\frac{\mu^2}{2\lambda} \equiv \frac{v^2}{2}$ if $\mu^2 < 0$, yielding a nonzero vacuum expectation value for the Higgs field. Expansion of ϕ around this minimum will yield the fermion and gauge boson mass terms and the Higgs interaction terms.

- The Higgs kinetic term and coupling to the $SU(2)_{EW} \times U(1)_Y$ gauge bosons, again following minimal coupling:

$$\begin{aligned} \mathcal{L}_{\phi^\dagger \phi, \phi^\dagger V^2 \phi} &= (D_\mu \phi)^\dagger (D^\mu \phi) && (3) \\ &= (\partial_\mu \phi)^\dagger (\partial^\mu \phi) && (4) \\ &\quad + \phi^\dagger \left(i\alpha_1 \frac{Y_\phi}{2} B_\mu + i\alpha_2 \frac{\tau_a}{2} W^a_\mu \right)^\dagger \left(i\alpha_1 \frac{Y_\phi}{2} B^\mu + i\alpha_2 \frac{\tau^a}{2} W^\mu_a \right) \phi \end{aligned}$$

  The cross terms involving $\phi^\dagger \partial_\mu \phi$ and its complex conjugate can be shown to vanish (see Huang [5], p. 110).

- The Higgs coupling to the fermions. These are known as Yukawa couplings, combining a Higgs scalar field ϕ with a left-handed fermion ψ_L ($SU(2)$ doublet) and a right-handed fermion ψ_R ($SU(2)$ singlet).

$$\begin{aligned} \mathcal{L}_{Yukawa} = & \left[(\mathbf{h}_E)_{jk}\, \phi\, \overline{l_{L,j}}\, e_{R,k} + (\mathbf{h}^\dagger_E)_{jk}\, \phi^\dagger\, \overline{e_{R,j}}\, l_{L,k} \right] \\ & + \left[(\mathbf{h}_U)_{jk}\, \phi^\dagger\, \overline{q_{L,j}}\, u_{R,k} + (\mathbf{h}^\dagger_U)_{jk}\, \phi\, \overline{u_{R,j}}\, q_{L,k} \right] \\ & + \left[(\mathbf{h}_D)_{jk}\, \phi\, \overline{q_{L,j}}\, d_{R,k} + (\mathbf{h}^\dagger_D)_{jk}\, \phi^\dagger\, \overline{d_{R,j}}\, q_{L,k} \right] \end{aligned} \tag{5}$$

  The $\mathbf{h}$ matrices are 3×3 complex matrices with indices running along generations. These matrices provide for both fermion masses and intergenerational mixing when the Higgs acquires its vacuum expectation value. Note that there is no neutrino term because we take neutrinos to be massless. Note also that ϕ and $\phi^\dagger$ are in reverse order in the u-quark term because of the need for the product to have hypercharge 0.

We now break electroweak symmetry. This consists of picking a vacuum expectation value for the Higgs field and expanding around that minimum. We choose

$$\phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix} \tag{6}$$

The vev must be chosen to lie along the neutral component of the $SU(2)_{EW}$ doublet in order to maintain a neutral vacuum. In addition, because ϕ^0 is complex, we must choose a phase for the vev; we simply take this to be zero, so the vev is real. Any phase could have been chosen and eliminated by a phase rotation.

The result is that the Higgs-fermion interaction terms breaks into two pieces. The first, given by replacing ϕ by its vacuum expectation value, gives terms of the form, for example, $\overline{l_L} l_R + \overline{l_R} l_L$ – *i.e.*, fermion mass terms. These introduce interactions between the left- and right-handed fermions of a given flavor – until now, the left- and right-handed fermions did not have any direct interactions with one another. The remaining piece yields interaction terms containing a Higgs scalar, a left-handed fermion, and a right-handed fermion. Note that, because the **h** matrices are non-diagonal, the Higgs vev yields intergenerational mixing. The reader may be more familiar with a picture in which intergenerational mixing appears via the Cabibbo-Kobayashi-Maskawi matrix that is a coefficienct of the electroweak charged current terms – *i.e.*, the interactions of the fermions with the $W^{\pm}$. The mixing matrices appear in the Higgs terms here because we chose a basis in which the fermions are eigenstates of the electroweak interaction. By applying a rotation

$$d_L \rightarrow d'_L = \mathbf{A}_L^{-1} d_L \tag{7}$$

$$d_R \rightarrow d'_R = \mathbf{A}_R^{-1} d_R \tag{8}$$

$$u_L \rightarrow u'_L = \mathbf{B}_L^{-1} u_L \tag{9}$$

$$u_R \rightarrow u'_R = \mathbf{B}_R^{-1} u_R \tag{10}$$

$$\mathbf{h}_D \rightarrow \mathbf{h}'_D = \mathbf{A}_L \mathbf{h}_D \mathbf{A}_R^{-1} \tag{11}$$

$$\mathbf{h}_U \rightarrow \mathbf{h}'_U = \mathbf{B}_L \mathbf{h}_U \mathbf{B}_R^{-1} \tag{12}$$

we can make the Higgs terms diagonal and make the charge current terms non-diagonal; this is equivalent to rotating to a mass-eigenstate basis. A similar rotation can be performed to the lepton sector. With massless neutrinos, the rotation is in fact trivial and introduces no lepton mixing.

3 The Minimal Supersymmetric Standard Model

We define the Minimal Supersymmetric Standard Model, or MSSM, in the following. It is the minimal supersymmetric extension to the Standard Model; by "minimal", we mean that it requires the smallest increase in particle content consistent with being invariant under SUSY transformations (modulo the soft SUSY-breaking and EWSB terms). For an understanding of *why* this is the minimal extension, please see Drees [1].

Just as a brief summary, supersymmetry transformations can be thought of as the "square root" of Lorentz transformations. The reason for this is that the anticommutator of two supersymmetry generators is the momentum operator, which generates space-time translations. Because of this, supersymmetry should be thought of more as a space-time symmetry (like Lorentz invariance) rather than as a gauge symmetry. Models invariant under global (independent of space-time coordinate) supersymmetry transformations do not introduce new gauge fields, just as invariance under global Lorentz transformations do not introduce new gauge fields.

3.1 Particle Content

There are only three irreducible representations of the combined supersymmetry and Lorentz groups: left- and right-handed chiral superfields and vector superfields. Left-handed chiral superfields contain a single complex scalar field and a single (complex) left-handed Weyl fermion (Weyl, for our purposes, just means two-component). The fermions and scalars of the Standard Model are taken to be components of left-handed chiral superfields. For right-handed chiral superfields, the Weyl fermion is right handed. Left- and right-handed chiral superfields exhibit a one-to-one correspondence. We choose to use only left-handed chiral superfields because the algebra is simplified (see footnote 3 of [1]). Vector superfields contain a single hermitian spin-1 field and a single Weyl fermion. The gauge bosons of the Standard Model are taken to be components of vector superfields. Please refer to the appendix for a review of spinor algebra, the relationship between Dirac and Weyl fermions, and our sign and symbol conventions (we follow Haber and Kane [2]).

The correspondence of fermions to left-handed superfields requires a bit more explanation. Clearly, left-handed fermions can be components of left-handed superfields. However, to get the right-handed fermions into left-handed superfields, we must conjugate them first to make them left-handed. Therefore, while $\hat{l}_L$ is the left-handed superfield containing the fermion l_L and the scalar $\widetilde{l_L}$, we have that $\widehat{u^c_R}$ is the left-handed superfield containing the fermion u^c_R and the scalar $\widetilde{u^c_R}$. Some authors will write these as $\hat{u}^c_R$, u^c_R, and $\tilde{u}^c_R$, which is confusing – it appears that the $\hat{}$ or $\tilde{}$ operation takes place before $_R$ and ^{c}. We will use the former convention to be clear.

3.2 SUSY-Invariant Lagrangian

We begin by introducing the SUSY-invariant and electroweak-symmetry-preserving Lagrangian. We will later introduce SUSY- and electroweak-symmetry breaking terms.

- The fermion kinetic terms and gauge couplings are similar to the Standard Model version, with the modification that the fermions are now 2-component Weyl spinors rather than 4-component Dirac spinors. This is possible because the Dirac spinors in the Standard Model are always separated into left- and right-handed projections. A helicity projection has only 2 complex degrees of freedom, and so representation of a helicity projection by a 2-component Weyl spinor is allowed. When the particle is massless, there is no mixing between the left- and right-handed components, so the left- and right-handed components can actually be treated as separate particles represented by distinct 2-component Weyl spinors. We will therefore rewrite the fermion kinetic and gauge terms in this 2-component form. For lack of symbols, we will actually use the same symbols as used for the Dirac spinors. The L and R subscripts are now somewhat misleading, since all the fields are left-handed; they refer to the link between the left-handed superfields and the Standard Model particles. R will always appear with conjugation in order to enforce the left-handedness requirement. See the appendix for definition of $\overline{\sigma}$ and for the explicit definition of the various spinor products below.

$$
\begin{aligned}
\mathcal{L}_{\overline{f}f,\overline{f}Vf} \quad = \quad \sum_j \quad \Big[\quad & -i\,\overline{l_{L,j}}\,\overline{\sigma}^\mu \left(\partial_\mu - ig_1 \tfrac{Y_{l_{L,j}}}{2} B_\mu - ig_2 \tfrac{\tau_a}{2} W^a_\mu\right) l_{L,j} \\
& -i\,\overline{e^c_{R,j}}\,\overline{\sigma}^\mu \left(\partial_\mu - ig_1 \tfrac{Y_{e_{R,j}}}{2} B_\mu\right) e^c_{R,j} \\
& -i\,\overline{q_{L,j}}\,\overline{\sigma}^\mu \left(\partial_\mu - ig_1 \tfrac{Y_{q_{L,j}}}{2} B_\mu - ig_2 \tfrac{\tau_a}{2} W^a_\mu\right) q_{L,j} \\
& -i\,\overline{u^c_{R,j}}\,\overline{\sigma}^\mu \left(\partial_\mu - ig_1 \tfrac{Y_{u_{R,j}}}{2} B_\mu\right) u^c_{R,j} \\
& -i\,\overline{d^c_{R,j}}\,\overline{\sigma}^\mu \left(\partial_\mu - ig_1 \tfrac{Y_{d_{R,j}}}{2} B_\mu\right) d^c_{R,j} \qquad \Big]
\end{aligned}
\tag{13}
$$

It should be noted that the use of $\overline{\sigma}$ differs from Drees' use of σ; we believe Drees is using a confusing shorthand, while the above usage is completely unambiguous (see the appendix for a discussion of this).

- We introduce the sfermion-fermion-gaugino interaction terms, as they are prescribed by extension of minimal coupling. It must be remembered that the terms in the Lagrangian must combine an even number of fermions (and any number of scalars) to ensure that the result is bosonic. The result is

$$
\begin{aligned}
\mathcal{L}_{\widetilde{f}\widetilde{V}f} \quad = \quad \sum_j \quad \Big[\quad & ig_1\sqrt{2}\left[\widetilde{l}_{L,j}^{\,\dagger} \tfrac{Y_{l_{L,j}}}{2}\widetilde{B}\,l_{L,j} - \tfrac{Y_{l_{L,j}}}{2}\,\overline{\widetilde{B}}\,\overline{l_{L,j}}\,\widetilde{l}_{L,j}\right] \\
& + ig_2\sqrt{2}\left[\widetilde{l}_{L,j}^{\,\dagger} \tfrac{\tau_a}{2}\widetilde{W}^a\,l_{L,j} - \tfrac{\tau_a}{2}\overline{\widetilde{W}^a}\,\overline{l_{L,j}}\,\widetilde{l}_{L,j}\right] \\
& + ig_1\sqrt{2}\left[\widetilde{e^c}_{R,j}^{\,\dagger} \tfrac{Y_{e_{R,j}}}{2}\widetilde{B}\,e^c_{R,j} - \tfrac{Y_{e_{R,j}}}{2}\,\overline{\widetilde{B}}\,\overline{e^c_{R,j}}\,\widetilde{e^c}_{R,j}\right] \\
& + ig_1\sqrt{2}\left[\widetilde{q}_{L,j}^{\,\dagger} \tfrac{Y_{q_{L,j}}}{2}\widetilde{B}\,q_{L,j} - \tfrac{Y_{q_{L,j}}}{2}\,\overline{\widetilde{B}}\,\overline{q_{L,j}}\,\widetilde{q}_{L,j}\right] \\
& + ig_2\sqrt{2}\left[\widetilde{q}_{L,j}^{\,\dagger} \tfrac{\tau_a}{2}\widetilde{W}^a\,q_{L,j} - \tfrac{\tau_a}{2}\overline{\widetilde{W}^a}\,\overline{q_{L,j}}\,\widetilde{q}_{L,j}\right] \\
& + ig_1\sqrt{2}\left[\widetilde{u^c}_{R,j}^{\,\dagger} \tfrac{Y_{u_{R,j}}}{2}\widetilde{B}\,u^c_{R,j} - \tfrac{Y_{u_{R,j}}}{2}\,\overline{\widetilde{B}}\,\overline{u^c_{R,j}}\,\widetilde{u^c}_{R,j}\right] \\
& + ig_1\sqrt{2}\left[\widetilde{d^c}_{R,j}^{\,\dagger} \tfrac{Y_{d_{R,j}}}{2}\widetilde{B}\,d^c_{R,j} - \tfrac{Y_{d_{R,j}}}{2}\,\overline{\widetilde{B}}\,\overline{d^c_{R,j}}\,\widetilde{d^c}_{R,j}\right] \quad \Big]
\end{aligned}
\tag{14}
$$

Each line contains a term analogous to the fermion-gauge boson-fermion terms of Equation 13 and its hermitian conjugate. One would expect the gaugino and fermion terms to be reversed after conjugation; they are returned to their original order and the sign changed to compensate. The above may appear complicated, but note how the same structure repeats, modulated by the possible absence of the $\widetilde{W}$ couplings in the case of the $SU(2)_{EW}$ singlet fields. The $\overline{f}$ fields have been replaced by $\widetilde{f}^\dagger$, as one might expect. The reader should recall that the "left-handed" fields (in the cases of the sfermions $\widetilde{l}_{L,j}$ and $\widetilde{q}_{L,j}$, this name is misleading since the particles themselves have no handedness; "left-handed" simply indicates their partnership with the Standard Model left-handed fermions) are always $SU(2)_{EW}$ doublets, as for the corresponding fermionic fields. Thus the $SU(2)_{EW}$ structure of the above exactly matches that of the fermionic kinetic/gauge Lagrangian. In terms containing a sfermion, the spin structure is carried by the gaugino term and the remaining fermion term. Note that the gaugino-sfermion-fermion interaction strengths are governed by exactly the same couplings as the corresponding gauge boson-fermion-fermion interactions, modulo arithmetic factors. This is a manifestation of supersymmetry.

- Kinetic terms and gauge coupling terms for the sfermions also are produced via the minimal coupling prescription (this will not necessarily be obvious – see Drees for a better understanding). The $U(1)_Y$ and $SU(2)_{EW}$ charges are the same as for the fermionic partners, as given, for example, in $\mathcal{L}_{\widetilde{f}\widetilde{V}f}$. The first piece is what one should expect for a scalar field; the second set of terms, containing g and g^2, comes from some gory details of SUSY that need not be explained here ($\widetilde{f}$ is taken to represent any sfermion):

$$\begin{aligned}
\mathcal{L}_{\widetilde{f}^\dagger\widetilde{f},\widetilde{f}^\dagger V\widetilde{f},\widetilde{f}^\dagger V^2\widetilde{f},\widetilde{f}^\dagger\widetilde{f}\widetilde{f}^\dagger\widetilde{f}} &= \left(D_\mu\widetilde{f}\right)^\dagger\left(D^\mu\widetilde{f}\right) - \frac{1}{2}g^2\left(\widetilde{f}^\dagger T^a\widetilde{f}\right)\left(\widetilde{f}^\dagger T_a\widetilde{f}\right) && (15)\\
&= \left(\partial_\mu\widetilde{f}\right)^\dagger\left(\partial^\mu\widetilde{f}\right) && (16)\\
&\quad - igV_\mu^a\left[\left(\partial^\mu\widetilde{f}^\dagger\right)T_a\widetilde{f} - \widetilde{f}^\dagger\left(T_a\partial^\mu\widetilde{f}\right)\right]\\
&\quad + g^2V_\mu^aV_b^\mu\widetilde{f}^\dagger T^aT_b\widetilde{f}\\
&\quad -\frac{1}{2}g^2\left(\widetilde{f}^\dagger T^a\widetilde{f}\right)\left(\widetilde{f}^\dagger T_a\widetilde{f}\right)
\end{aligned}$$

  where V_μ^a is a gauge boson, T_a are the generators for the gauge group, and g is the gauge coupling. These terms should of course be summed over gauge groups and sfermions; we will not bother to write these out explicitly for all gauge fields and fermions. The above indicate the types of couplings one gets. The full gory expansions (in the mass-eigenstate basis) are given in Jungman, Kamionkowski, and Griest, eqns. A.34 and A.35. Note that this term introduces a $|\widetilde{f}|^4$ term in the scalar potentials for the sfermions and that these terms are fully determined by the gauge groups and couplings.

- Kinetic and gauge interaction terms for the gauginos are produced by an unobvious SUSY-invariant prescription. The gauge boson kinetic terms are as in the Standard Model.

$$\begin{aligned}
\mathcal{L}_{V\,kin,\widetilde{V}\overline{\widetilde{V}},\widetilde{V}V\overline{\widetilde{V}}} &= -\tfrac{1}{4}F_{\mu\nu}^aF_a^{\mu\nu}\\
&\quad + \left(-\tfrac{i}{2}\widetilde{V}^a\sigma_\mu\partial^\mu\overline{\widetilde{V}_a} + \tfrac{1}{2}gf^{abc}\widetilde{V}_a\sigma_\mu V_b^\mu\overline{\widetilde{V}_a} + h.c.\right)
\end{aligned} \qquad (17)$$

  The use of σ here instead of $\overline{\sigma}$ is again a detail of spinor algebra; please see the appendix. In this case, however, we match Drees' convention.

- The two Higgs fields. A second Higgs field must be introduced to keep the Yukawa terms supersymmetric (see the discussion of $calL_{Yukawa}$. Since the Higgs fields are left-chiral superfields, just like the superfields containing the Standard Model fermions, the minimal coupling prescription yields similar kinetic and interaction terms. We can basically copy all the terms from Equations 13, 14, and 16 and put in the appropriate gauge fields and couplings:

$$\begin{aligned}
\mathcal{L}_{Higgs} \;=\; & -i\,\overline{\widetilde{H}_1}\,\overline{\sigma}^{\mu}\left(\partial_{\mu} - ig_1\tfrac{Y_{H_1}}{2}B_{\mu} - ig_2\tfrac{\tau_a}{2}W^a_{\mu}\right)\widetilde{H}_1 \\
& + ig_1\sqrt{2}\left[H_1^{\dagger}\tfrac{Y_{H_1}}{2}\widetilde{B}\,\widetilde{H}_1 - \tfrac{Y_{H_1}}{2}\,\overline{\widetilde{B}}\,\overline{\widetilde{H}_1}\,H_1\right] \\
& + ig_2\sqrt{2}\left[H_1^{\dagger}\tfrac{\tau_a}{2}\widetilde{W}^a\,\widetilde{H}_1 - \tfrac{\tau_a}{2}\overline{\widetilde{W}^a}\,\overline{\widetilde{H}_1}\,H_1\right] \\
& + (\partial_{\mu}H_1)^{\dagger}(\partial^{\mu}H_1) \\
& - ig_1B_{\mu}\left[\left(\partial^{\mu}H_1^{\dagger}\right)\tfrac{Y_{H_1}}{2}\;H_1 - H_1^{\dagger}\left(\tfrac{Y_{H_1}}{2}\;\partial^{\mu}H_1\right)\right] \\
& - ig_2W^a_{\mu}\left[\left(\partial^{\mu}H_1^{\dagger}\right)\tfrac{\tau_a}{2}\;H_1 - H_1^{\dagger}\left(\tfrac{\tau_a}{2}\;\partial^{\mu}H_1\right)\right] \\
& + g_1^2B_{\mu}B^{\mu}H_1^{\dagger}\left(\tfrac{Y_{H_1}}{2}\right)^2 H_1 \\
& + g_2^2W^a_{\mu}W^{\mu}_bH_1^{\dagger}\tfrac{\tau^a}{2}\tfrac{\tau_b}{2}H_1 \\
& - \tfrac{1}{2}g_1^2\left(H_1^{\dagger}\left(\tfrac{Y_{H_1}}{2}\right)^2 H_1\right)\left(H_1^{\dagger}\left(\tfrac{Y_{H_1}}{2}\right)^2 H_1\right) \\
& - \tfrac{1}{2}g_2^2\left(H_1^{\dagger}\tfrac{\tau^a}{2}H_1\right)\left(H_1^{\dagger}\tfrac{\tau_a}{2}H_1\right) \\
& + \mu^2H_1^{\dagger}H_1 \\
& + H_1 \rightarrow H_2 \\
& - \left[\mu\widetilde{H}_1\widetilde{H}_2 + h.c.\right]
\end{aligned} \tag{18}$$

The first line gives the Higgsino kinetic and gauge interaction terms. The next two lines are the Higgs-gaugino-Higgsino interaction terms, analogous to Equation 14. The fourth line is the Higgs scalar kinetic term. The fifth through eighth lines are the Higgs scalar-gauge boson interactions. The ninth and tenth lines are the gauge contributions to the Higgs scalar potential, giving quartic terms. The eleventh line is the Higgs scalar mass. The thirteenth line gives the Higgsinos mass and mixes them. These last two μ terms are added by hand to help give rise to the Higgs scalar potential needed for electroweak symmetry breaking; it should be noted that, as written, these terms are in fact supersymmetric and gauge invariant.

- Finally, the Yukawa couplings. There will be a piece that is corresponds directly to the Standard Model Higgs-fermion-fermion couplings that yield the Standard Model particle masses via the Higgs mechanism. One change is necessary: $\phi^{\dagger}$ in the Standard Model u-quark mass term must be replaced by a new Higgs field $\widehat{H}_2$ because terms of the form $\widehat{H}^{\dagger}\widehat{fermion}_L\widehat{fermion}^c_R$ are not supersymmetric because $\widehat{H}^{\dagger}$ is a right-handed chiral superfield (see Drees [1] for a deeper explanation of this). In addition, there will be new Higgsino-fermion-sfermion terms. These are necessary to maintain supersymmetry: a pure Higgs scalar-fermion-fermion turns into a Higgsino-fermion-sfermion term under supersymmetry transformations. Again, see Drees for the specific way in which these are generated.

$$
\begin{aligned}
\mathcal{L}_{Yukawa} \;=\; & -\left[(\mathbf{h}_E)_{jk}\, H_1\, l_{L,j}\, e_k + (\mathbf{h}_E^\dagger)_{jk}\, \overline{e_{R,j}}\, \overline{l_{L,k}}\, H_1^\dagger\right] \\
& -\left[(\mathbf{h}_U)_{jk}\, H_2\, q_{L,j}\, u_k + (\mathbf{h}_U^\dagger)_{jk}\, \overline{u_{R,j}}\, \overline{q_{L,k}}\, H_2^\dagger\right] \\
& -\left[(\mathbf{h}_D)_{jk}\, H_1\, q_{L,j}\, d_k + (\mathbf{h}_D^\dagger)_{jk}\, \overline{d_{R,j}}\, \overline{q_{L,k}}\, H_1^\dagger\right] \\
& -\left[(\mathbf{h}_E)_{jk}\, \widetilde{H}_1\, l_{L,j}\, \widetilde{e^c_{Rk}} + (\mathbf{h}_E^\dagger)_{jk}\, \widetilde{e^c_{Rk}}^\dagger\, \overline{l_{L,j}}\, \overline{\widetilde{H}}_1\right] \\
& -\left[(\mathbf{h}_U)_{jk}\, \widetilde{H}_2\, q_{L,j}\, \widetilde{u^c_{Rk}} + (\mathbf{h}_U^\dagger)_{jk}\, \widetilde{u^c_{Rk}}^\dagger\, \overline{q_{L,j}}\, \overline{\widetilde{H}}_2\right] \\
& -\left[(\mathbf{h}_D)_{jk}\, \widetilde{H}_1\, q_{L,j}\, \widetilde{d^c_{Rk}} + (\mathbf{h}_D^\dagger)_{jk}\, \widetilde{d^c_{Rk}}^\dagger\, \overline{q_{L,j}}\, \overline{\widetilde{H}}_1\right] \\
& -\left[(\mathbf{h}_E)_{jk}\, \widetilde{H}_1\, e^c_{R,k}\, \widetilde{l_{L,j}} + (\mathbf{h}_E^\dagger)_{jk}\, \widetilde{l_{L,j}}^\dagger\, \overline{e^c_{R,k}}\, \overline{\widetilde{H}}_1\right] \\
& -\left[(\mathbf{h}_U)_{jk}\, \widetilde{H}_2\, u^c_{R,k}\, \widetilde{q_{L,j}} + (\mathbf{h}_U^\dagger)_{jk}\, \widetilde{q_{L,j}}^\dagger\, \overline{u^c_{R,k}}\, \overline{\widetilde{H}}_2\right] \\
& -\left[(\mathbf{h}_D)_{jk}\, \widetilde{H}_1\, d^c_{R,k}\, \widetilde{q_{L,j}} + (\mathbf{h}_D^\dagger)_{jk}\, \widetilde{q_{L,j}}^\dagger\, \overline{d^c_{R,k}}\, \overline{\widetilde{H}}_1\right]
\end{aligned} \tag{19}
$$

There is not much to be said here, except that one has to be careful with taking the hermitian conjugate to generate the second terms in each line – there are spinor indices, $SU(2)_{EW}$ indices, and generation indices to be properly matched up.

So we have fully supersymmetrized the Standard Model Lagrangian with, in fact, a decrease in the number of free parameters: the two Standard Model Higgs parameters μ and λ have been traded for a single parameter μ. λ is provided by the gauge couplings of the Higgs to the gauge fields and is therefore not free.

4 Supersymmetry- and Electroweak Symmetry-Breaking

The above Lagrangian is a) Supersymmetric and b) electroweak symmetric. Nature is neither. So this Lagrangian does not match reality and must be modified.

Soft supersymmetry-breaking terms are introduced. "Soft" has a technical meaning – these are the only terms that can break SUSY while introducing no uncancellable quadratically diverging diagrams. The following are the most general set of soft breaking terms given the particle content of the MSSM (Girardello and Grisaru [7] have demonstrated this). They are as follows:

$$
\begin{aligned}
\mathcal{L}_{soft} \;=\; & \widetilde{q_L}^\dagger \mathbf{M^2}_q \widetilde{q_L} + \widetilde{u^c_R}^\dagger \mathbf{M^2}_u \widetilde{u^c_R} + \widetilde{d^c_R}^\dagger \mathbf{M^2}_d \widetilde{d^c_R} + \widetilde{l_L}_\dagger \mathbf{M^2}_l \widetilde{l_L} + \widetilde{e^c_R}^\dagger \mathbf{M^2}_e \widetilde{e^c_R} \\
& + \left(\mathbf{A}_e \mathbf{h}_e H_1 \widetilde{l_L} \widetilde{e^c_R} + \mathbf{A}_u \mathbf{h}_u H_2 \widetilde{q_L} \widetilde{u^c_R} + \mathbf{A}_d \mathbf{h}_d H_1 \widetilde{q_L} \widetilde{d^c_R} + h.c.\right) \\
& + (B\mu H_1 H_2 + h.c) \\
& + m_1^2 H_1^\dagger H_1 + m_2^2 H_2^\dagger H_2 \\
& + \frac{1}{2} M_1 \widetilde{B}\widetilde{B} + \frac{1}{2} M_2 \widetilde{W}_a \widetilde{W}^a + \frac{1}{2} M_3 \widetilde{g}_a \widetilde{g}^a
\end{aligned} \tag{20}
$$

The first row are scalar mass terms: these additional terms give the sfermions different masses (and mixings) than their fermion partners. Note that the coefficients $\mathbf{M^2}$ are matrices in 3×3 generation space. They should not be taken to be the product of two matrices $\mathbf{M}$ but rather a single matrix $\mathbf{M^2}$. The second row introduces new Higgs couplings for the sfermions (also 3×3 in generation space); in addition to affecting the sfermion masses, these new couplings also allow the flavor (CKM) mixing of the sfermions to be different from that of their fermionic partners. The third row modifies the Higgs mixing term for the scalar Higgs only, distinguishing the Higgs scalars from their fermionic superpartners. Additional scalar Higgs mass terms are introduced in the fourth row. Finally, gaugino mass terms are introduced to break the gauge boson-gaugino mass equality. The gauginos belonging to a given gauge group are required to have the same mass to avoid breaking the gauge symmetries.

Discuss electroweak symmetry breaking.

5 Renormalization and Running Masses and Couplings

Too damn lazy to write this yet.

6 MSSM Free Parameters

We discuss all the parameters that may or may not be modified in minimal supersymmetric extensions to the Standard Model.

6.1 Fixed Parameters

Parameters fixed by experiment:

- **h**: The quark and lepton mass and mixing matrices. Fixed by observed Standard Model particle masses and mixing.
- g_1, g_2, g_3: These gauge couplings are fixed by the observed strengths of the electroweak and strong interaction..
- $v_1^2 + v_2^2$: This quantity is fixed by the W and Z masses.

6.2 Free Parameters

The parameters that are free are:

- $\mathbf{M^2}_{\widetilde{f}}$: The sfermion masses. These are 3×3 hermitian matrices. At least five of them exist $\rightarrow$ 45 parameters. Note that these may be nondiagonal and therefore may themselves contribute to sfermion mixing, though we have fully counted this freedom already.
- **A**: The sfermion mixing matrices. These are three 3×3 matrices with no hermiticity requirement. There are therefore 18 free parameters per matrix, yielding a total of 54 free parameters. There is a loose requirement that the minima of all the sfermions'

scalar potentials be minimized at 0, but that only puts some allowed range on the matrix components; it does not yield specific relations between components.

- M_1, M_2, M_3: The gaugino masses. Three free parameters.
- $m_1^2, m_2^2, B, \tan\beta \equiv \frac{v_2}{v_1}, \mu$: Higgs parameters (supersymmetry and electroweak symmetry breaking). m_1^2 and m_2^2 are real scalars. B is a scalar and may be complex. $\tan\beta$ is limited by experiment from below and by m_t/m_b from above. Of these, we can choose three to be free and determine the others via the electroweak symmetry breaking relations. Choices of parameter sets are
  - $m_1^2, m_2^2, \tan\beta$ free, B and μ dependent (Drees, Equation 69).
  - $B, \mu, and \tan\beta$ free, m_1^2 and m_2^2 dependent (Drees, Equation 68).
  - $m_A^2 = -\frac{2B\mu}{\sin 2\beta}, \tan\beta, \mu$ free, B, m_1^2, and m_2^2 dependent. (m_A^2 is the mass of the pseudoscalar Higgs boson). We have introduced a new non-independent quantity m_A^2, so there are now three dependent parameters instead of two. The reason for this choice is that m_A^2 is a more physical parameter. This is what is used in most numerical codes. It should be noted that these conditions hold even for radiative electroweak symmetry breaking – radiative EWSB imposes an additional condition, but leaves these relations valid.

7 Frameworks

We now discuss some of the many supersymmetric frameworks in use. By "framework", we mean a set of requirements on the various free parameters. This is distinguished from a "model", which is a specific choice of all the parameters, yielding a mass spectrum and allowing calculation of cross-sections, etc.. The frameworks are of varying strictness; these serve to constrain the huge parameter space available. We begin with the most constrained framework and work to less constrained ones.

7.1 mSUGRA

Short for *minimal supergravity* – this framework assumes that, above the Planck scale (10^{19} GeV), supersymmetry can be gauged: the invariance under global supersymmetry transformations can be promoted to invariance under local supersymmetry transformations. This will yield general relativity, since supersymmetry transformations are related to Lorentz transformations. Some mechanism breaks this local invariance at energies below the Planck scale. However, since the GUT scale (10^{16} GeV) is so much lower, there's no reason to expect this mechanism to distinguish between the different gauge groups and particle generations. Thus, mSUGRA yields the maximal set of assumptions:

- Gaugino mass unification: $M_1 = M_2 = M_3 \equiv m_{1/2}$ at M_{GUT}.
- Fermion Yukawa coupling unification: $\mathbf{h}_E = \mathbf{h}_U = \mathbf{h}_D \equiv \mathbf{h}$ at M_{GUT}.
- Sfermion mass and mixing unification: $\mathbf{M^2}_{\tilde{l}} = \mathbf{M^2}_{\tilde{q}} = \mathbf{M^2}^2_{\tilde{e}} = \mathbf{M^2}_{\tilde{u}} = \mathbf{M^2}_{\tilde{d}} = m_1^2\mathbf{1} = m_2^2\mathbf{1} \equiv m_0^2\mathbf{1}$ and $\mathbf{A}_e = \mathbf{A}_u = \mathbf{A}_d \equiv A_0\mathbf{1}$ at M_{GUT}.

- $B = A_0 + m_0$ at M_{GUT}.
- Some specification of the GUT gauge group. This is necessary to determine running of couplings from Planck scale down to the GUT scale.
- Radiative electroweak symmetry breaking. This provides another relationship between μ^2 and the other Higgs sector parameters, thereby adding another condition and reducing the number of free parameters in the Higgs sector to two (usually taken to be m_A^2, the pseudoscalar mass, and $\tan\beta$.

This framework is the most constraining one available. It is not used, however, because some of its consequences violate observations. These will be noted below as we discuss more free frameworks.

7.2 mSUGRA$_{\mathrm{LHC}}$/mSUGRA$_{\mathrm{LEP}}$

This is what people mean when they discuss LEP data or LHC expectations in the context of "minimal supergravity". A few assumptions are relaxed:

- Yukawa coupling unification is not required. It is impossible to get the 1st and 2nd generation couplings to unify at the GUT scale and still yield reasonable masses at observed energies. So people usually release the 1st and 2nd generation coupling unification immediately. mSUGRA$_{\mathrm{LHC}}$/mSUGRA$_{\mathrm{LEP}}$ also release the 3rd generation – this is also known as "no b/τ mass unification" for obvious reasons.
- Radiative electroweak symmetry breaking is employed, but, when picking models, no checks are made on the color/charge neutrality of the vacuum for the given set of parameters.

7.3 Generic SUGRA

Releases assumptions further:

- No Higgs-sfermion mass unification. The two Higgs masses are assumed to be unified, $m_1^2 = m_2^2$. It is usually m_A^2 that is specified, which is related to these by EWSB. All the sfermion masses are assumed to be unified, $\mathbf{M^2}_{\tilde{l}} = \mathbf{M^2}_{\tilde{q}} = \mathbf{M^2}_{\tilde{e}} = \mathbf{M^2}_{\tilde{u}} = \mathbf{M^2}_{\tilde{d}} \equiv m_0^2\mathbf{1}$, but no requirement is made that the two match. That is, $m_{1,2}^2 \neq m_0^2$ at M_{GUT} is allowed.
- No radiative electroweak symmetry breaking – we rely on standard spontaneous electroweak symmetry breaking, so there are three free Higgs parameters as discussed earlier.

The release of Higgs mass unification is done for three reasons. First, it allows one to separate the Higgs and sfermion sectors (the sleptons and squarks are linked by the above relation at the GUT scale), sending one or the other to very high mass to simplify analysis of models. Second, the gauge couplings do not unify properly at M_{GUT} if Higgs-sfermion mass unification is enforced. Finally, superstring theories suggest there really is no connection between the Higgs and sfermion masses.

7.4 Constrained MSSM

Make some parameters "quasi-free":

- $\mathbf{A}$ and $\mathbf{m}_{\tilde{f}}$ matrices are diagonal (sfermion mixing same as fermion mixing) but otherwise unconstrained. People frequently forget to specify in which basis this constraint is made, leading to much confusion. Frequently, they will also take only A_t and A_b nonzero since these dominate the sfermion mass effects.
- Higgs-sfermion mass unification not required (as is necessary if sfermion mass unification not required!)

7.5 MSSM

No constraints, not even gaugino mass unification.

8 Neutralino Sector

In this section we summarize the neutralino sector of the MSSM, emphasising its significance in cold dark matter search.

8.1 Mass Matrix and Mixing

The $SU(2)_{EW} \times U(1)_Y$ part of the MSSM includes B, W_3, H_1^0, H_2^0 and their super-partners, wino, bino and two higgsinos. The super-partners have the same quantum numbers (spin 1/2, no charge etc) and they form what is referred to as the neutralino sector. Since these fields have the same quantum numbers, they can mix. In other words, their mass matrix can have non-diagonal entries. The physical states are obtained by rotating the fields (i.e. mixing them), so that the mass matrix becomes diagonal.

The part of the MSSM which is invariant under supersymmetry (see Section 3.2) transformation contributes only one parameter to this matrix, namely μ from the $\mu H_1 H_2$ term. However, when the soft supersymmetry breaking terms (see Section 4) are added, the matrix becomes more complex. In general, the matrix is given by (in $\widetilde{B}$, $\widetilde{W}$, $\widetilde{H}_1^0$, $\widetilde{H}_2^0$ space)

$$\mathbf{M} = \begin{pmatrix} M_1 & 0 & -m_Z \sin\theta_W \cos\beta & +m_Z \sin\theta_W \sin\beta \\ 0 & M_2 & +m_Z \cos\theta_W \cos\beta & -m_Z \cos\theta_W \sin\beta \\ -m_Z \sin\theta_W \cos\beta & +m_Z \cos\theta_W \cos\beta & \delta_{33} & -\mu \\ -m_Z \sin\theta_W \sin\beta & -m_Z \cos\theta_W \sin\beta & -\mu & \delta_{44} \end{pmatrix} \tag{21}$$

where $\tan\beta = v_2/v_1$ is the ratio of vacuum expectation values for the two Higgs bosons, $M_{1,2}$ are the gaugino masses (introduced by the SUSY breaking terms) and m_Z is the mass of the Z boson. The δ's are radiative corrections important when two higgsinos are close in mass.

The eigenvalues of this matrix are the masses of the 4 physical neutralino states. The neutralino states are then the eigenvectors and they are given by:

$$\chi^0 = N_{i1}\widetilde{B} + N_{i2}\widetilde{W}_3 + N_{i3}\tilde{H}_1^0 + N_{i4}\tilde{H}_2^0 \tag{22}$$

where N's are the mixing coefficients, chosen so that the neutralino masses are non-negative.

Since the nondiagonal entries (except for $-\mu$ entries) are of the order of m_Z (i.e. small compared to 1 TeV scale), they can be neglected in many cirumstances. Therefore, it is relatively easy to analyze the eigenvalues of this matrix:

1. If $M_{1,2}$ are large ($\sim$1 TeV or larger) and μ is small (i.e., $\sim m_Z$), then the neutralino of smallest mass would tend to be higgsino-like (that is, $N_{i3,4}$ would be larger than $N_{i1,2}$).

2. If μ is large and $M_{1,2}$ are not, then the lightest neutralino would tend to be gaugino-like.

3. If μ and $M_{1,2}$ are similar in magnitude and large ($>> m_Z$), the lightest neutralino would be a relatively even mixture of higgsinos and gauginos.

4. If μ and $M_{1,2}$ are similar in magnitude and small (m_Z), the lightest neutralino is a complicated mixture that is sensitive to the value of $\tan\beta$.

These situations are shown schematically on Figure 1. It is clear that the parameters M_1, M_2, μ and to a smaller extent $\tan\beta$ directly determine the WIMP candidate.

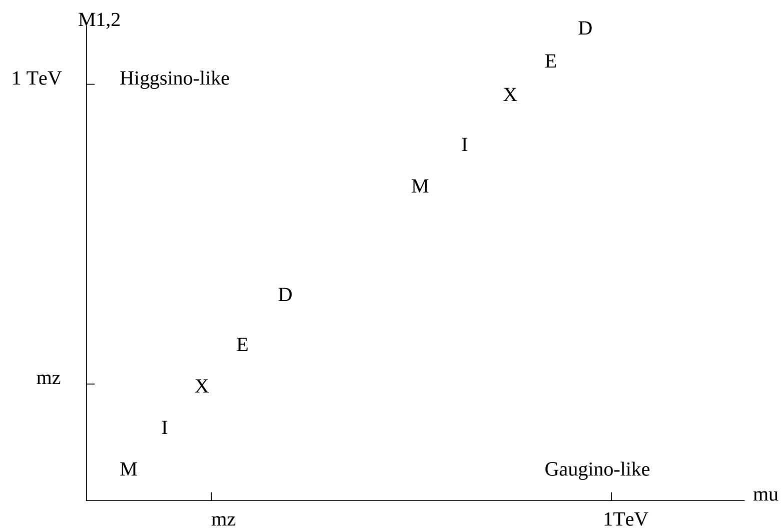

Figure 1: Different neutralino models in M_2 vs μ plane

A common constraint on the soft SUSY breaking parameters is the unification of gaugino masses at Grand Unified Theory (GUT) scale (see Section 7 for which frameworks use this constraint; it also used in P. Gondolo's DarkSUSY software [8]). However, the renormalization group equation (see Section 5) for gaugino masses implies $M_1/M_2 = \alpha_1/\alpha_2$, where α_i's are the appropriate running couplings. The consequence is that $M_1 < M_2$ everywhere below the GUT scale, so the lightest gaugino at the weak scale (of the order of m_Z) is usually bino.

8.2 Neutralino-Nucleon Scattering Cross-Section Expectations

In direct detection experiments, three processes can contribute to neutralino-quark scattering: Higgs exchange (spin-independent), Z exchange (spin-dependent), and squark exchange (spin-independent and spin-dependent). Because of this, one can show that pure bino or higgsino neutralino states have lower scattering cross-sections than mixed states:

- Higgs exchange: only $\widetilde{B}\widetilde{H}H$ (i.e. bino, higgsino, higgs) vertex is allowed (other combinations, $\widetilde{B}\widetilde{B}H$ and $\widetilde{H}\widetilde{H}H$, are 0 since there are no corresponding terms in the Lagrangian - it is enough to consider the Standard Model equivalent combinations to see this). This suppresses bino and higgsino interactions via Higgs (and mixed is favored).
- Z exchange: $\widetilde{B}\widetilde{B}Z$, $\widetilde{H}\widetilde{B}Z$ and $\widetilde{H}^0_1\widetilde{H}^0_2 Z$ are 0, but $\widetilde{H}^0_i\widetilde{H}^0_i Z$ is not (for the last vertex, need to expand the higginos in real and imaginary parts, it is not enough to just consider the equivalent combinations in the Standard Model). This suppresses the bino and mixed state interactions via Z.

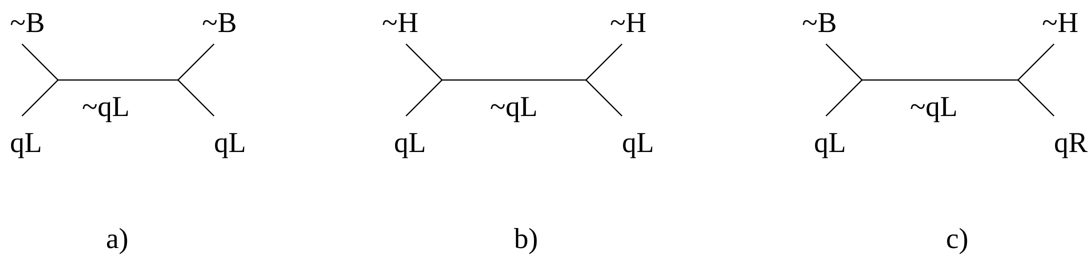

Figure 2.

Figure 2:

- Squark exchange: interactions like a) and b) on Figure 2 are allowed, but are spin-dependent (i.e. the amplitudes corresponding to these Feynman diagrams are proportional to the spin, so they are suppressed). On the other hand, interaction c) on Figure 2 is spin independent (the amplitude is proportional to the mass of the quark; the $\overline{q}_L q_R$ term is the quark mass term). This difference results from details of the γ-matrix algebra, but can basically be traced to the fact that a) and b) have q_L as incoming and outgoing particles, but c) has q_L and q_R. Therefore, the spin-dependent interactions of the bino and the higgsino via squarks are again suppressed with respect to the mixed states. Note: here it is implicitly assumed that there is no mixing of L and R squarks. This assumption can be relaxed, in which case processes $\widetilde{B}q_L \rightarrow \widetilde{B}q_R$ and $\widetilde{H}q_L \rightarrow \widetilde{H}q_R$ (and their time-reverses) become possible. Since these are spin independent, the interactions of bino (or higgsino) with nuclei via squark exchange are no longer suppressed.

It follows that every channel of interaction between a neutralino and a nucleon is suppressed if the neutralino is a pure bino or pure higgsino. As an aside, we note that, if the neutralino is pure higgsino, then the spin-dependent cross-section is larger than in mixed or

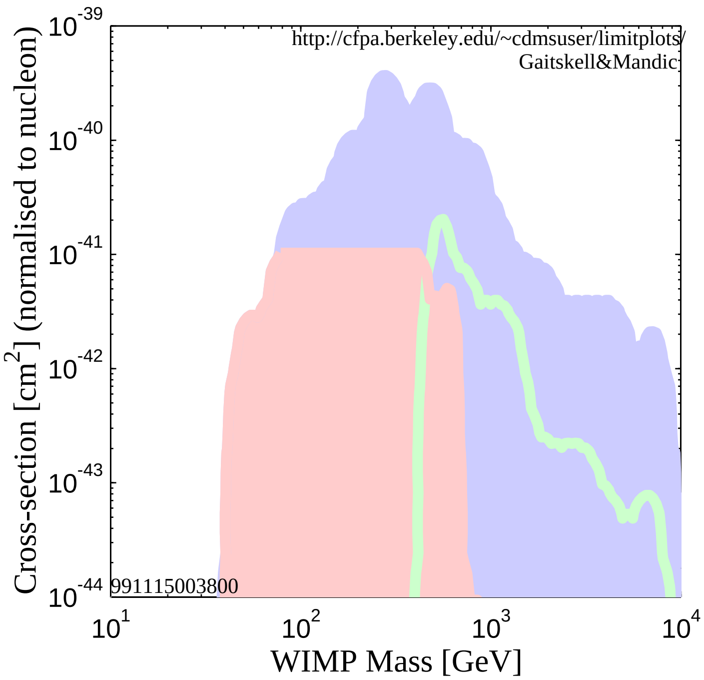

Figure 3: The largest region corresponds to mixed neutralino, the smaller region on the left corresponds to bino and the smaller region on the right corresponds to higgsino.

pure bino cases because Z exchange is allowed; however, the spin-dependent cross-section is still likely to be smaller than the spin-independent cross-section. The size of the pure bino/higgsino suppression effect can be seen at
http://cfpa.berkeley.edu/∼cdmsuser/limitplots
as well as in Figure 3. The effect is about one order of magnitude - the most recent results are already probing the bino and mixed models.

The allowed region for the WIMP on this plot (in all models) is determined by the following:

- The upper bound on the WIMP mass comes from the requirement that $\Omega h^2 < 1$, where Ω is the relic density of the WIMP and h is the Hubble parameter divided by 100 km s^{-1} Mpc^{-1}.

- The requirement that $\Omega h^2 > 0.025$ limits σ from above and it is also causing the discontinuity in the higgsino region.

- There may be annihilation processes that greatly suppress the WIMP relic density, more than one might expect. For example, for a higgsino, when the WIMP mass exceeds M_W, annihilation to W pairs via Higgs becomes possible and greatly increases the annihilation cross-section and suppresses the relic density. If the relic density drops below the lower cutoff, no models will appear on the plot – this explains the empty swath for higgsino models with masses near M_W.

- The accelerator results bound the mass of the WIMP from below.
- The lower bound on σ probably comes from the way the parameter space is parsed, although this is not certain yet.

It may be surprising that the scattering cross-section can be so low. The textbook WIMP argument uses the fact that, in order to have a significant relic density, WIMPs must have an annihilation cross-section of the order of the weak scale. The crossing argument is used to show that the WIMP-quark elastic-scattering cross-section is of the same order of magnitude – essentially, the same diagrams, rotated by 90 degrees, give the two cross-sections. This argument implicitly assumes that annihilation to quarks is a "typical" annihilation process: annihilation to, for example, quarks and W's should have roughly the same amplitudes, with allowances for different numbers of final states. But if the quark diagrams are especially suppressed, then the quark annihilation channel and hence the WIMP-quark scattering cross-section are suppressed, while the relic density is left unchanged because other annihilation processes keep the annihilation cross-section at the weak scale.

A Spinor Algebra

We will begin by describing the algebraic rules for Weyl spinors and then define Dirac spinors in terms of them. We will demonstrate how the Lagrangian for a massless Dirac fermion reduces to the Lagrangian for two uncoupled Weyl fermions, the left- and right-handed projections. We will gloss over most of the interesting details of how left- and right-handed Weyl spinors are different representations of the inhomogeneous Lorentz symmetry group, blah blah blah blah. Refer to Haber and Kane [2], Wess and Bagger [6], Bagger [3], or Ryder [4] for that discussion. We note that our space-time metric is

$$g_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1) \tag{23}$$

A.1 Weyl Spinors – Definition and Manipulation

There are two kinds of 2-component Weyl spinors, left- and right-handed ones. These are the $(\mathbf{\frac{1}{2}}, \mathbf{0})$ and $(\mathbf{0}, \mathbf{\frac{1}{2}})$ representations of $SL(2, C)$, the inhomogeneous Lorentz symmetry group, which is simply the set of all space-time rotations and boosts (the homogeneous Lorentz group) with the addition of all space-time translations. These facts are not important as far as we are concerned. We simply state our conventions and a number of rules to be followed for manipulating them:

- The left-handed spinors you are used to from elementary quantum mechanics are written as

  $$\xi_\alpha \tag{24}$$

  with $\alpha = 1, 2$. These are written as 2-component column vectors.

- Right-handed spinors are written as

  $$\overline{\xi_{\dot\alpha}} \tag{25}$$

  with $\dot\alpha = 1, 2$. Note the dot symbol over the index. Dotted indices and undotted indices are distinct; you may not contract a dotted index with an undotted index. Right-handed Weyl spinors are related to left-handed spinors via conjugation:

  $$\overline{\xi_{\dot\alpha}} \equiv \xi^*_\alpha \tag{26}$$

  These are also written as two component column vectors. The distinction between unbarred and barred spinors (undotted and dotted indices) is that they obey different transformation rules under $SL(2, C)$; they belong to different representations, and thus can not be directly mixed.

- Indices may be raised or lowered using the completely antisymmetric 2×2 matrix $\epsilon^{\alpha\beta}$:

  $$\begin{aligned} \epsilon^{\alpha\beta} &\equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} & (27) \\ &\equiv -\epsilon_{\alpha\beta} \\ &= -\epsilon^{\beta\alpha} \\ &= \epsilon_{\beta\alpha} & (28) \end{aligned}$$

ϵ makes no distinction between dotted and undotted indices; dotted indices may be switched to undotted indices and vice versa at will. This allows us to define left- and right-handed spinors with raised indices:

$$\xi^\alpha \equiv \epsilon^{\alpha\beta}\xi_\beta \tag{29}$$

$$\overline{\xi^{\dot\alpha}} \equiv \epsilon^{\dot\alpha\dot\beta}\overline{\xi_{\dot\alpha}} \tag{30}$$

Raised-index spinors are thus quite different from lowered index spinors; they in fact obey different $SL(2,C)$ transformation rules than the corresponding lowered-index spinors. We will need them in order to form products of spinors that are invariant under $SL(2,C)$ (*i.e.*, Lorentz and rotationally invariant).

- We define the 2×2 σ matrices in the standard way:

$$\begin{array}{ll} \sigma^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} & \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} & \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \end{array} \tag{31}$$

i.e.

$$\sigma^\mu = (\mathbf{1}, \vec\sigma) \tag{32}$$

The two indices of σ^μ are not equivalent; σ^μ is written as

$$\sigma^\mu_{\alpha\dot\alpha} \tag{33}$$

That is, σ^μ is always written with lowered indices and with the second index dotted. In effect, this means that σ^μ may only be multiplied on the left by raised-index left-handed (undotted-index) spinors ξ^α and on the right by raised-index right-handed (dotted-index) spinors $\overline{\xi^{\dot\alpha}}$. We define $\overline{\sigma}^\mu$ by

$$\overline{\sigma}^{\mu\dot\alpha\alpha} = \epsilon^{\dot\alpha\dot\beta}\epsilon^{\alpha\beta}\sigma^\mu_{\beta\dot\beta} \tag{34}$$

Note the order of the indices! It is not intuitive. If you are not careful, you can pick up a -1. $\overline{\sigma}^\mu$ is basically the version of σ^μ with indices raised and the dotted and undotted indices interchanged. Analogous to σ^μ, $\overline{\sigma}^\mu$ may only be multiplied on the left by lowered-index right-handed (dotted-index) spinors $\overline{\xi_{\dot\alpha}}$ and on the right by lowered-index left-handed (undotted-index) spinors ξ_α. Using this definition, one can show

$$\overline{\sigma}^\mu = (\mathbf{1}, -\vec\sigma) \tag{35}$$

You have to expand out each component in μ separately to see this, but it is straightforward.

- Now that we know about the rules for indices, we can write down the multiplication rules. Authors rarely write down the spinor indices in supersymmetry articles, so these rules are needed to know what is meant by a product of two Weyl spinors or Weyl spinors and σ matrices. Note that when we exchange the order of two spinors, after they have been written in component form (*e.g.*, $\eta^\alpha \xi_\alpha = -\xi_\alpha \eta^\alpha$), we pick up -1; this has nothing to do with the matrix algebra, but rather is based on the fact that these spinors are actually fermionic field operators and, as such, they anticommute (*e.g.*, going from the second to the third expression on the first line, or the antipenultimate (see Appendix B) to the penultimate position on the second line):

$$\begin{aligned}
\eta\,\xi \equiv \quad \eta^\alpha \xi_\alpha \quad &= -\,\xi_\alpha \eta^\alpha \qquad (36)\\
&= \eta^\alpha \epsilon_{\alpha\beta}\,\xi^\beta = \epsilon_{\alpha\beta}\,\eta^\alpha \xi^\beta = -\epsilon_{\beta\alpha}\,\eta^\alpha \xi^\beta = -\eta_\beta\,\xi^\beta = \xi^\beta \eta_\beta = \xi\eta\\
\overline{\eta}\,\overline{\xi} \equiv \quad \overline{\eta_{\dot\alpha}}\,\overline{\xi^{\dot\alpha}} \quad &= -\,\overline{\xi^{\dot\alpha}}\,\overline{\eta_{\dot\alpha}} \qquad (37)\\
&= \overline{\eta_{\dot\alpha}}\,\epsilon^{\dot\alpha\dot\beta}\,\overline{\xi_{\dot\beta}} = \epsilon^{\dot\alpha\dot\beta}\,\overline{\eta_{\dot\alpha}}\,\overline{\xi_{\dot\beta}} = -\epsilon^{\dot\beta\dot\alpha}\,\overline{\eta_{\dot\alpha}}\,\overline{\xi_{\dot\beta}} = -\overline{\eta^{\dot\beta}}\,\overline{\xi_{\dot\beta}} = \overline{\xi_{\dot\beta}}\,\overline{\eta^{\dot\beta}} = \overline{\xi}\,\overline{\eta}
\end{aligned}$$

Be careful to note the difference in definition of $\eta\,\xi$ and $\overline{\eta}\,\overline{\xi}$ in terms of the positions of the indices. Note that expressions of the form $\eta\,\overline{\xi}$ and $\overline{\eta}\,\xi$ are not allowed because the indices do not work out correctly: these would require summing over a pair of indices consisting of one dotted and one undotted index. For those who want more of a justification, it would require multiplication of spinors that are in different representations of $SL(2,C)$, which clearly makes no sense.

We also define products involving σ matrices:

$$\begin{aligned}
\overline{\eta}\,\overline{\sigma}^\mu\,\xi \equiv \quad \overline{\eta_{\dot\alpha}}\,\overline{\sigma}^{\mu\dot\alpha\beta}\,\xi_\beta \quad &= \quad -\,\xi_\beta\,\overline{\sigma}^{\mu\dot\alpha\beta}\,\overline{\eta_{\dot\alpha}} && \equiv -\,\xi\,\overline{\sigma}^\mu\,\overline{\eta}\\
&= \quad \overline{\eta_{\dot\alpha}}\,\epsilon^{\dot\alpha\dot\gamma}\,\epsilon^{\beta\delta}\,\sigma^\mu_{\delta\dot\gamma}\,\xi_\beta && = \left(-\epsilon^{\dot\gamma\dot\alpha}\right)\,\overline{\eta_{\dot\alpha}}\,\sigma^\mu_{\delta\dot\gamma}\left(-\epsilon^{\delta\beta}\right)\,\xi_\beta\\
& && = \eta^{\dot\gamma}\,\sigma^\mu_{\delta\dot\gamma}\,\xi^\delta\\
& && \equiv \overline{\eta}\,\sigma^\mu\,\xi \qquad (38)\\
&= \quad -\,\xi_\beta\,\epsilon^{\dot\alpha\dot\gamma}\,\epsilon^{\beta\delta}\,\sigma^\mu_{\delta\dot\gamma}\,\overline{\eta_{\dot\alpha}} && = -\left(-\epsilon^{\delta\beta}\right)\,\xi_\beta\,\sigma^\mu_{\delta\dot\gamma}\left(-\epsilon^{\dot\gamma\dot\alpha}\right)\,\overline{\eta_{\dot\alpha}}\\
& && = -\,\xi^\delta\,\sigma^\mu_{\delta\dot\gamma}\,\eta^{\dot\gamma}\\
& && \equiv -\,\xi\,\sigma^\mu\,\overline{\eta}
\end{aligned}$$

The definitions of $\overline{\eta}\,\overline{\sigma}^\mu\,\xi$ and $\xi\,\sigma^\mu\,\overline{\eta}$ are pretty much what you would expect based on the index structure of the components. The other two forms, $\xi\,\overline{\sigma}^\mu\,\overline{\eta}$ and $\overline{\eta}\,\sigma^\mu\,\xi$ are unexpected and, in my opinion, should never be used. However, Drees [1] does indeed use these last two forms for his fermion Lagrangian, and so readers should be aware.

A.2 Weyl Fermion Lagrangian

We now write down the Lagrangian for Weyl fermions of both kinds. We give no particular justification, but we will see that these are consistent with the Lagrangian for the Dirac fermion composed of two Weyl fermions.

The Lagrangian for a left-handed Weyl fermion ξ is

$$\begin{aligned}
\mathcal{L}_L \quad &= \quad -i\overline{\xi}\,\overline{\sigma}^\mu \partial_\mu \xi = -i\overline{\xi_{\dot\alpha}}\,\overline{\sigma}^{\mu\dot\alpha\beta}\partial_\mu \xi_\beta \qquad (39)\\
&= \quad i\left(\partial_\mu \overline{\xi}\right)\,\overline{\sigma}^\mu \xi \qquad (40)
\end{aligned}$$

where we have integrated by parts and dropped a total derivative in going from the first line to the second. The Lagrangian for a right-handed Weyl fermion $\overline{\eta}$ is

$$\begin{aligned}\mathcal{L}_R &= -i\eta\,\sigma^\mu\partial_\mu\overline{\eta} = -i\eta^\alpha\,\sigma^\mu_{\alpha\dot{\beta}}\partial_\mu\overline{\eta^{\dot{\beta}}} \qquad (41)\\ &= i\overline{\eta}\,\overline{\sigma}^\mu\partial_\mu\eta \\ &= i\left(\partial_\mu\eta\right)\,\sigma^\mu\overline{\eta} \qquad (42)\end{aligned}$$

again integrating the first line by parts to get the third line. The second line uses the relation between "natural" σ products from Equation 38. In both cases, we have used only the "natural" version of the σ products. It is clear here that the Lagrangian for a right-handed fermion $\overline{\eta}$ is the negative of the Lagrangian for the corresponding left-handed fermion η.

For completeness, we write down the Lagragians using the "unnatural" forms. These will no doubt confuse the reader further.

$$\begin{aligned}\mathcal{L}_L &= -i\,\overline{\xi}\,\sigma^\mu\partial_\mu\xi \qquad & \mathcal{L}_R &= -i\,\eta\,\overline{\sigma}^\mu\partial_\mu\overline{\eta} \\ &= i\left(\partial_\mu\overline{\xi}\right)\,\sigma^\mu\xi & &= i\left(\partial_\mu\eta\right)\,\overline{\sigma}^\mu\overline{\eta}\end{aligned} \qquad (43)$$

We use integration by parts to derive the second line from the first in each case.

A.3 Dirac-Weyl Correspondence and Dirac Fermion Lagrangian

Dirac spinors are calculated from Weyl spinors in a straigtforward way. We must work in the helicity basis for the Dirac spinors in order to make this simple. In this basis, we define a Dirac spinor ψ to be made of left- and right-handed Weyl spinors ξ and $\overline{\eta}$:

$$\psi = \begin{pmatrix} \xi_\alpha \\ \overline{\eta^{\dot{\alpha}}} \end{pmatrix} \qquad (44)$$

In this basis, the γ matrices are defined by

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu_{\alpha\dot{\beta}} \\ \overline{\sigma}^{\mu\dot{\alpha}\beta} & 0 \end{pmatrix} = \left((\begin{matrix} 0 & (\mathbf{1},\vec{\sigma})^\mu \\ (\mathbf{1},-\vec{\sigma})^\mu & 0 \end{matrix} \right) \qquad (45)$$

The next thing we need is $\overline{\psi} \equiv \psi^\dagger\gamma^0$:

$$\begin{aligned}\overline{\psi} &= \begin{pmatrix} \left(\xi^*_\alpha\right)^T & \left(\overline{\eta^{\dot{\alpha}}}^*\right)^T \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad (46)\\ &= \begin{pmatrix} \left(\overline{\eta^{\dot{\alpha}}}^*\right)^T & \left(\xi^*_\alpha\right)^T \end{pmatrix} \\ &= \begin{pmatrix} \left(\eta^\alpha\right)^T & \left(\overline{\xi_{\dot{\alpha}}}\right)^T \end{pmatrix}\end{aligned}$$

where in the last line we have used the component relationship between barred and unbarred Weyl spinors, Equation 26. We now write down the Lagrangian for a free, massless Dirac fermion, in the standard form:

$$
\begin{aligned}
\mathcal{L}_{Dirac} &= i\overline{\psi}\gamma^\mu\partial_\mu\psi \\
&= i\left(\begin{array}{cc} \left(\eta^\alpha\right)^T & \left(\overline{\xi_{\dot\alpha}}\right)^T \end{array} \right) \left(\begin{array}{cc} 0 & \sigma^\mu_{\ \alpha\dot\beta} \\ \overline{\sigma}^{\mu\dot\alpha\beta} & 0 \end{array} \right) \partial_\mu \left(\begin{array}{c} \xi_\beta \\ \overline{\eta^{\dot\beta}} \end{array} \right) \\
&= i\overline{\xi_{\dot\alpha}}\,\overline{\sigma}^{\mu\dot\alpha\beta}\partial_\mu\,\xi_\beta + i\eta^\alpha\,\sigma^\mu_{\ \alpha\dot\beta}\partial_\mu\,\overline{\eta^{\dot\beta}} \\
&= i\overline{\xi}\,\overline{\sigma}^\mu\partial_\mu\,\xi + i\eta\,\sigma^\mu\partial_\mu\,\overline{\eta}
\end{aligned} \tag{47}
$$

This is exactly the sum of $\mathcal{L}_L$ and $\mathcal{L}_R$, as defined in Equations 39 and 41, modulo a minus sign. We are still baffled as to the loss of this sign; it presumably is significant. Of course, if the Dirac fermion is a pure helicity state (left- or right-handed), one of the terms in the Lagrangian disappears. For the gauge interaction terms, ∂_μ is simply replaced by B_μ, $T_a W^a_\mu$, etc.

We also write down the translation of a Dirac mass term:

$$
\begin{aligned}
\mathcal{L}_{mass,Dirac} &= -m\,\overline{\psi}\,\psi \\
&= -m\,\eta^\alpha\,\xi_\alpha - m\,\overline{\xi_{\dot\alpha}}\,\overline{\eta^{\dot\alpha}}
\end{aligned} \tag{48}
$$

which clearly appears as an interaction between the left- and right-handed components of the Dirac spinor.

Thus, it is clear how to write the kinetic and gauge terms for the Standard Model fermions in minimal supersymmetry – one simply makes the correspondence between a left-handed Dirac fermion ψ_L and a left-handed Weyl fermion ξ and between a right-handed Dirac fermion ψ_R and a right-handed Weyl fermion $\overline{\eta}$. γ^μ is replaced by $\overline{\sigma}^\mu$ for left-handed Dirac fermions and by σ^μ for right-handed Dirac fermions. There remains this outstanding question of a sign flip; for now, we assume that the Weyl fermion Lagrangian expressions, Equations 39 and 41, are correct. Note that, since we use the conjugates of the right-handed fermions as the fundamental fermion fields in the MSSM, they too are left-handed. Thus, the Lagrangian for left-handed Weyl fermions is all we need to write the fermionic piece of the MSSM.

B Glossary

antipenultimate: the position just before penultimate
framework: a set of requirements on the various free MSSM parameters
model: a specific choice of all the free parameters, yielding a mass spectrum and allowing calculation of cross-sections, etc.

References

[1] M. Drees, NATO ASI

[2] Haber and Kane, Phys Rep

[3] J. Bagger, NATO ASI

[4] L. Ryder, QFT

[5] K. Huang, Quarks, Leptons, and Gauge Fields

[6] J. Wess and J. Bagger

[7] Ghirardello and Grisaru

[8] P. Gondolo, DarkSUSY documentation.